

UNDECIDABILITY RESULTS FOR SHUFFLE LANGUAGES^{1,2}

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ABSTRACT

We consider shuffle languages which are an extension of regular languages with two operators, that is shuffle (sometimes called interleaving) and shuffle closure. The class of shuffle languages is a proper subset of context-sensitive languages, incomparable with context-free ones. For this class we give two undecidability results. First of all we show that the intersection emptiness problem for shuffle languages is undecidable and then we prove that there is no algorithm to determine the shuffle closure height of shuffle expressions.

Keywords: Shuffle, shuffle closure, undecidable problems.

1. Introduction

The operations shuffle and shuffle closure were introduced to describe concurrencies, synchronization and cyclic actions. Together with other operations they describe various classes of languages which have been extensively studied (see [8, 9, 10, 11]). It is well known (see [6]) that the class of regular languages is closed under shuffle operation. On the other hand, if both operations of shuffle and shuffle iteration are added to regular operations, i. e. sum, concatenation and KLEENE closure we obtain the class of shuffle languages SL . This class is contained in the class of context-sensitive languages and incomparable with either context-free languages or PETRI net languages. On the other hand, if regular expressions are extended by four operators: shuffle, shuffle closure, synchronous composition and renaming (with no shuffle closure nested), then they generate precisely PETRI net languages, as was shown in [4].

As far as decidability questions for shuffle languages are concerned, at first it was proved [2] that the equality problem for shuffle languages is undecidable, and then [7] the undecidability of the universe problem, that is whether $L = \Sigma^*$, where L is a language and Σ an alphabet. Of course the former result follows from the latter.

¹Full version of a submission presented at the Second International Conference “Developments in Language Theory” DLT’95, Magdeburg, July 17–21, 1995.

²This research was supported by University of Gdańsk grant BW 5100-5-0230-5.