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CANTOR SETS AND DEJEAN'S CONJECTURE [1]

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ABSTRACT

Let $k \in \mathbb{R}$ be given, $1 < k < 2$. It is shown that for a large enough alphabet Σ, the set of ω-words over Σ avoiding powers greater than k is a CANTOR set. In particular, a new method for showing the existence of ω-words over Σ avoiding powers greater than k is given. This presents a new way in which to attack DEJEAN's conjecture. We also give a new non-constructive proof that words of length at least 2^n over an n letter alphabet are avoidable.

Keywords: DEJEAN's conjecture, CANTOR sets, words avoiding patterns, repetitive threshold.

1. Introduction

Infinite words avoiding certain patterns have been studied since the turn of the century [16]. The tools for studying such words have almost invariably been specially constructed substitutions. In this paper we develop an alternative tool for studying such words and are able to draw conclusions about the set of all those ω-words over a finite alphabet which avoid x^k, k some fixed real number between 1 and 2. It seems likely that this set, when non-empty, is always a *perfect* set; if w is any ω-word over Σ avoiding x^k, then we can find another ω-word v over Σ avoiding x^k, and agreeing with w for as long an initial segment as we wish. This claim has already been established for the case where k is a natural number [6, 7]. In this paper we extend our earlier work to cover cases where k is allowed to be fractional.

Much work has been done on the problem of showing the existence of a single ω-word over a given alphabet avoiding a given pattern [2, 3, 4, 13, 17]. This problem

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