Journal of Automata, Languages and Combinatorics **1** (1996) 1, 37–53

ON THE MODULARITY OF DECIDABILITY OF COMPLETENESS AND TERMINATION [1]

KAI SALOMAA
Department of Mathematics, University of Turku
FIN–20014 Turku, Finland
e-mail: ksalomaa@sara.cc.utu.fi

ABSTRACT

We consider decision problems for direct sums of term rewriting systems where the components belong to classes for which the corresponding property is decidable. We show that decidability of termination is not a modular property for finite left-linear monadic term rewriting systems. It is known that termination is not modular [28]. Our result generalizes this nonmodularity result by establishing that for given two terminating finite left-linear term rewriting systems we cannot even decide algorithmically whether or not their direct sum terminates. Decidability of confluence and decidability of completeness of (left- or right-) linear term rewriting systems are shown to be modular. Decidability of completeness of general systems is shown to be nonmodular.

Keywords: Term rewriting systems, modularity, decidability, POST correspondence problem.

1. Introduction

A property of term rewriting systems is called modular if it is preserved under disjoint union (or direct sum). Modular aspects of term rewriting systems have been first considered by TOYAMA [27] where he proved that the confluence property is modular. On the other hand, it was established quite early that the disjoint union of terminating systems in general does not terminate [28]. Sufficient conditions for modularity of termination were established by RUSINOWITCH [24] and MIDDELDORP [19]. These criteria are generalized and presented in a unified framework by GRAMLICH [9]. The property of having unique normal forms is shown to be modular by MIDDELDORP [18]. TOYAMA, KLOP and BARENDREGT [29] prove that completeness is modular for left-linear systems, and, MIDDELDORP and TOYAMA [22] establish the modularity of completeness for constructor systems. KURIHARA and OHUCHI [15] prove that the termination property is modular for systems where termination can be proved using a simplification ordering. KURIHARA and OHUCHI [16] generalize this result by showing that simple termination is preserved for combinations of systems that are allowed to have common function symbols which do not appear at the root of any left-hand

[1] This research has been supported by the Academy of Finland.