Journal of Automata, Languages and Combinatorics **1** (1996) 1, 13–26

OMEGA-SYNTACTIC CONGRUENCES [1]

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ABSTRACT

An ω-language over a finite alphabet X is a set of infinite sequences of letters of X. Previously studied syntactic equivalence relations defined by ω-languages have mainly been relations on X^*. In this paper the emphasis is put on relations in X^ω, by associating to an ω-language L a congruence on X^ω, called the ω-syntactic congruence of L. Properties of this congruence and notions induced by it, such as ω-residue, ω-density, and separativeness are defined and investigated. Finally, a congruence on X^* related to the ω-syntactic congruence and quasi-orders on X^ω induced by an ω-language are studied.

Keywords: ω-syntactic congruence, ω-language, dense language, disjunctive language, residue, syntactic monoid.

1. Introduction

Various types of congruences on X^* have been introduced in connection with ω-words and ω-languages. The usual equivalence relations induced by an ω-language L on X^* are R_L and P_L, defined by (see, for example, [6]):

$$w \equiv v(R_L) \Leftrightarrow (\forall y \in X^\omega,\ wy \in L \text{ iff } vy \in L)$$
$$w \equiv v(P_L) \Leftrightarrow (\forall x \in X^*,\ y \in X^\omega,\ xwy \in L \text{ iff } xvy \in L).$$

Both R_L and P_L are equivalence relations on X^* which coincide with the Nerode and syntactic equivalence if L is a language over X^*. One easily proves that R_L is a right congruence and that P_L is a congruence. The monoid $\mathrm{Syn}(L) = X^*/P_L$ is called the *syntactic monoid* of L.

An ω-language L is said to be disjunctive or right disjunctive if the corresponding relation P_L or R_L is the equality. It is dense if for every $u \in X^*$ there exist $x \in X^*$, $y \in X^\omega$ such that $xuy \in L$. Obviously, if an ω-language is disjunctive, it is dense. If the index of P_L is finite, then L is said to be μ-regular. (μ-regular ω-languages are sometimes referred to as finite-state ω-languages; see, for example, [7]) Remark that the index of P_L is finite if and only if the index of R_L is finite.

[1] This research was supported by Grant OGP0007877 of the Natural Sciences and Engineering Research Council of Canada