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# ON THE OPTIMALITY OF AN ALGORITHM OF REINGOLD AND SUPOWIT [1]

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ABSTRACT

In [5], REINGOLD and SUPOWIT have analyzed a divide–and–conquer heuristic to obtain a matching with a small cost. In this paper we show that — among a large class of similar strategies — the version of REINGOLD and SUPOWIT produces the minimal expected costs.

*Keywords:* Matching, cost, divide-and-conquer heuristic, average-case analysis, Rice's method.

Let $n$ be an even integer, and $P$ a set of $n$ points in the plane. A matching is a set of $n/2$ edges such that each point of $P$ is an endpoint of exactly one edge. The sum of the lengths of the edges is called the cost of the matching. In [5], REINGOLD and SUPOWIT have analyzed a divide-and-conquer heuristic to obtain a matching with a small cost. In two previous papers [6, 7] they have performed a worst-case analysis of several minimal matching algorithms.

Here, we want to demonstrate, that — among a large class of similar strategies — the version of REINGOLD and SUPOWIT produces the minimal expected costs.

For the sake of shortness, we assume a certain knowledge of [5].

REINGOLD and SUPOWIT start with a rectangle with sides $\sqrt{2}$ and 1. Then, it is bisected vertically to form two rectangles of sides 1 and $1/\sqrt{2}$, respectively. If both rectangles contain an even number of points, the matching is constructed in both rectangles separately. If both rectangles contain an odd number of points, the recursive strategy leaves one point in each rectangle (the "stranded point"), and the two lonely points are connected.

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